Analysis of a Wide Resonant Strip in Waveguide

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***Abstract*—An analysis has been developed to calculate the susceptance due to a wide resonant strip on the transverse plane of a rectangular waveguide. The theory is based upon a variational form for the susceptance, where the current distribution is determined by the moment method. Accurate results have been obtained for the wide strips as confirmed by the measurements.**

I. Introduction

Waveguide strip discontinuities have many applications in filters, tuning circuits, and impedance matching networks. Although the inductive strips have been used for many of these applications, capacitive (or resonant) strips have the advantage of providing a large variation in susceptance. Almost any value of susceptance can be obtained at a specified operating frequency by properly choosing the strip depth and strip width.

Resonant strip and probe discontinuities in a rectangular waveguide have been extensively studied [1]–[4]. These analyses were all concentrated on a narrow strip (or probe) and a constant current distribution was generally assumed. The multifilament moment method has solved the problem of a probe discontinuities in a semi-infinite waveguide [5], and inductive obstacles [6], [7] in rectangular waveguide. A general approach for solving inductive obstacles has been found in [8]. The wide resonant strip case has received little attention.

This paper reports an analysis for a wide resonant strip based on the variational method and the moment method. The analysis accommodates strips that are wide and could be off-centered on the transverse plane of the waveguide. Theoretical results have been compared with experiments and the agreement is good.

II. Formulation

The structure to be analyzed is shown in Fig. 1(a). The strip is located at $z = 0$. It is assumed to be infinitesimally thin and perfectly conducting, but not necessarily narrow in the transverse plane. The circuit can be represented by a shunt susceptance $\bar{B}$ as given in Fig. 1(b). Using the requirement that the tangential E field vanishes on the perfectly conducting strip surface S, we have the following

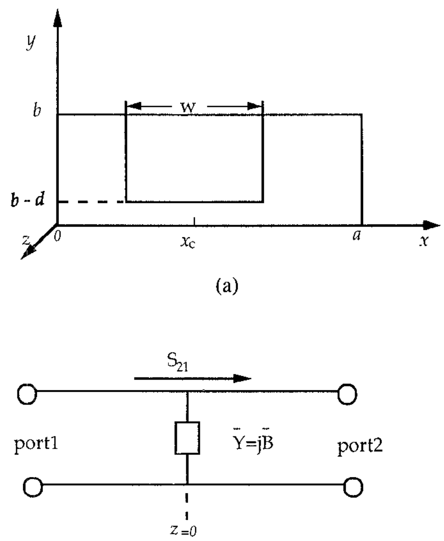

Fig. 1. A rectangular waveguide with a wide resonant strip. (a) Cross section. (b) Equivalent circuit.

integral equation

$$\sin\frac{\pi x}{a} - j\omega\mu_0 \int_S [G_y(r|r')]_{z=0} J_y(x', y')\, dx'\, dy' = 0 \tag{1}$$

where $J_y(x', y')$ is the current distribution on the strip, a is the width of the rectangular waveguide and the integral is performed over the strips surface S. Since the incident field is the dominant TE_{10} mode with E_y and H_x field components, only the y-directed current is considered here because the transverse electric current is small. G is the Green's function right at the cross section where the strip lies. It is explicitly written as follows [1]:

$$[G_y(r|r')]_{z=0} = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \cdot \frac{(2-\delta_m)(k_0^2 - m^2\pi^2/b^2)}{abk_0^2\Gamma_{nm}} \cdot \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi x'}{a}\right) \cdot \cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{m\pi y'}{b}\right) \tag{2}$$

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where

$$\Gamma_{nm} = \left(\frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{a^2} - k_0^2\right)^{1/2}$$

and

$$\delta_m = \begin{cases} 1, & \text{when } m = 0; \\ 0, & \text{otherwise.} \end{cases}$$

The current in integral equation (1) is chosen to be

$$J_y(x, y) = \sum_{q=1}^{L_x} I_q \sin\left[k_1(y - b + d)\right] \cdot \left[U(x - x_q) - U(x - x_{q+1})\right] \quad (3)$$

where $k_1 = \pi/2d$, L_x is the total number of segments in x direction, I_q is the current in the qth segment, and $U(x)$ is the unit step function. This form of current is more general than that used in [1], [9]. A similar approach has been successfully applied to cases of inductive strips [10].

When the current in (3) is substituted into the integral equation (1), the integral equation becomes

$$\sum_q I_q \left\{ \frac{(\cos k_1 d - 1)}{k_1} \sum_{n=1}^{\infty} \frac{1}{n\Gamma_{n0}} \cdot \left(\cos\frac{n\pi x_{q+1}}{a} - \cos\frac{n\pi x_q}{a}\right) \sin\frac{n\pi x}{a} + \frac{2k_1}{k_0^2} \sum_{n=1}^{\infty} \frac{1}{n} \left(\cos\frac{n\pi x_{q+1}}{a} - \cos\frac{n\pi x_q}{a}\right) \cdot \sin\frac{n\pi x}{a} A_n \right\} = \frac{\pi b}{j\omega\mu_0} \sin\frac{\pi x}{a} \quad (4)$$

where

$$A_n = -\sum_{m=1}^{\infty} \frac{1}{\Gamma_{nm}} \frac{(k_0 + m\pi/b)(k_0 - m\pi/b)}{(k_1 + m\pi/b)(k_1 - m\pi/b)} \cdot \cos\frac{m\pi(b - d)}{b} \cos\frac{m\pi y}{b}.$$

The chosen weighting functions have the same form of the expansion function; that is, the Galerkin method is adopted. Explicitly, the weighting functions are of the following form:

$$w_p(x, y) = \sin\left[k_1(y - b + d)\right]\left[U(x - x_p) - U(x - x_{p+1})\right] \quad p = 1, 2, \cdots, L_x \quad (5)$$

Inner product of (4) and (5) over the strip yields

$$\sum_q I_q \left\{ \frac{1}{k_1^2} \sum_{n=1}^{\infty} \frac{1}{n^2\Gamma_{n0}} \left(\cos\frac{n\pi x_{q+1}}{a} - \cos\frac{n\pi x_q}{a}\right) \cdot \left(\cos\frac{n\pi x_{p+1}}{a} - \cos\frac{n\pi x_p}{a}\right) + \frac{2k_1^2}{k_0^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot \left(\cos\frac{n\pi x_{q+1}}{a} - \cos\frac{n\pi x_q}{a}\right) \cdot \left(\cos\frac{n\pi x_{p+1}}{a} - \cos\frac{n\pi x_p}{a}\right) B_n \right\} = \frac{-\pi b}{j\omega\mu_0} \frac{1}{k_1} \left(\cos\frac{\pi x_{p+1}}{a} - \cos\frac{\pi x_p}{a}\right) \quad (6)$$

where

$$B_n = \sum_{m=1}^{\infty} \frac{1}{\Gamma_{nm}} \frac{(k_0 + m\pi/b)(k_0 - m\pi/b)}{[(k_1 + m\pi/b)(k_1 - m\pi/b)]^2} \cdot \cos^2\left(\frac{m\pi(b - d)}{b}\right)$$

This is a matrix equation for current coefficients I_q's. After solving I_q's, we use them in the variational formula to calculate the normalized susceptance. A variational analysis has been derived by Chang and Khan [1] to calculate the circuit parameter $\overline{B}$ with the following form:

$$\overline{B} = -2j \frac{k_0^2}{\Gamma_{10}} \frac{\left[\int_S J_y \sin\frac{\pi x}{a}\, dx\, dy\right]^2}{\left(\sum_{n=2}^{\infty}\sum_{m=0}^{\infty} + \sum_{m=1;n=1}^{\infty}\right) \frac{1}{\Gamma_{nm}} (2 - \delta_m)(k_0 + m\pi/b)(k_0 - m\pi/b) J_{nm}} \quad (7)$$

where

$$J_{nm} = \left[\int_S J_y \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b}\, dx\, dy\right]^2$$

The variational form for susceptance, (7), can be rearranged as

$$\overline{B} = -2j \frac{k_0^2}{\Gamma_{10}} \frac{\left[\sum_q I_q \left(\cos\frac{\pi x_{q+1}}{a} - \cos\frac{\pi x_q}{a}\right)\right]^2}{2\sum_{n=1}^{\infty} D_n E_n + k_0^2 \sum_{n=2}^{\infty} E_n/\Gamma_{n0}} \quad (8)$$

where

$$D_n = \sum_{m=1}^{\infty} \frac{(k_0 + m\pi/b)(k_0 - m\pi/b)}{\Gamma_{nm}} \cdot \left(\frac{k_1^2 \cos\frac{m\pi(b - d)}{b}}{(k_1 + m\pi/b)(k_1 - m\pi/b)}\right)^2$$

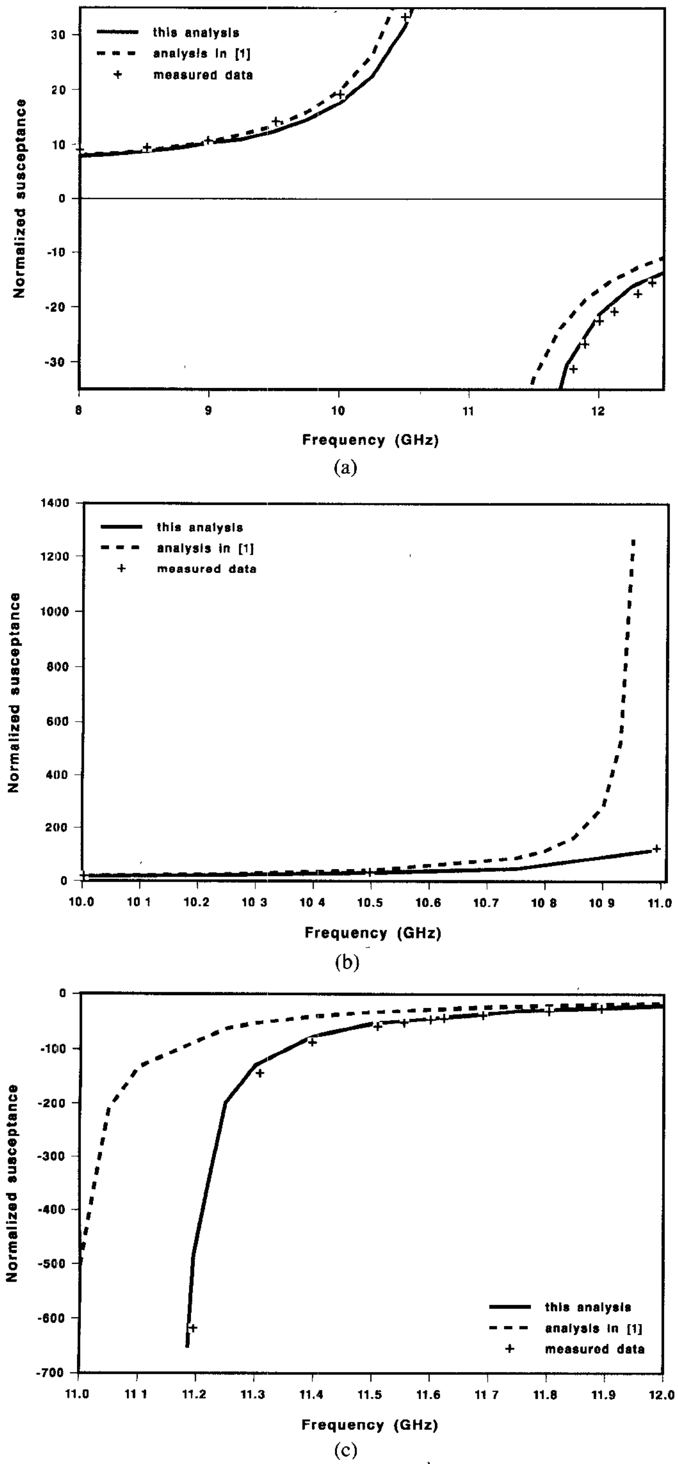

Fig. 2. (a) Normalized susceptance of a centered strip with $w = 0.280$ inch and $d = 0.365$ inch. (b) and (c) are more detailed plots near resonance.

and

$$E_n = \left[\frac{1}{n} \sum_q I_q \left(\cos \frac{n\pi x_{q+1}}{a} - \cos \frac{n\pi x_q}{a} \right) \right]^2$$

In general, the current coefficients I_q's are complex numbers. In our case, since the current is on a transverse plane, all the current coefficients have the same phase. Since current appears both in the numerator and denominator of the variational formula, the constant phase term of current is cancelled accordingly. Therefore only the magnitude of the current is needed in the numerical computation to save CPU time.

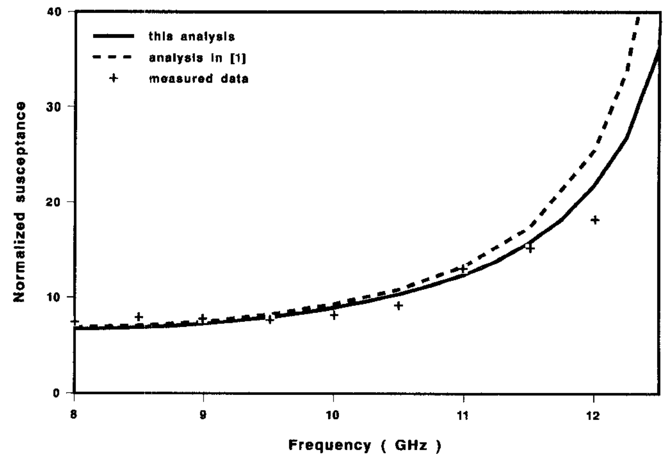

Fig. 3. Normalized susceptance of a centered strip with $w = 0.410$ inch and $d = 0.360$ inch.

III. Results

Experiments were carried out in X-band to verify the theory. TRL method and HP8510 network analyzer were used to measure S_{21}. The normalized susceptance is calculated by the following relationship [10]:

$$\bar{B} = -\frac{2 \sin \phi}{|S_{21}|} \tag{8}$$

where ϕ is the phase of S_{21}.

A. Centered Strips

Two cases have been checked to verify the analysis. They are the centered strips: case 1 of width = 0.280 inch and depth = 0.365 inch, and case 2 of width = 0.410 inch and depth = 0.360 inch. Fig. 2(a) shows the results for case 1. Fig. 2(b) and (c) give more detailed plots near resonance. The agreement between this analysis and measurements is better than the results using the narrow-strip analysis in [1], especially near resonance, where the value of susceptance is large. Results for the second case are shown in Fig. 3. It can be seen that both analyses agree with the measured data in the region where the susceptance is low, but this analysis gives better agreement with the measurements near resonance, where the value of susceptance is larger.

B. Off-Centered Strips

Measurements for off-centered strips have also been carried out for a strip centered at 0.315 inch ($0.35a$) and 0.405 inch ($0.45a$) with a width 0.266 inch and depth 0.3543 inch. Comparisons between the experimental and the theoretical results are shown in Figs. 4 and 5. Again, the agreement between this analysis and measurements is better than the results using the narrow-strip analysis in [1].

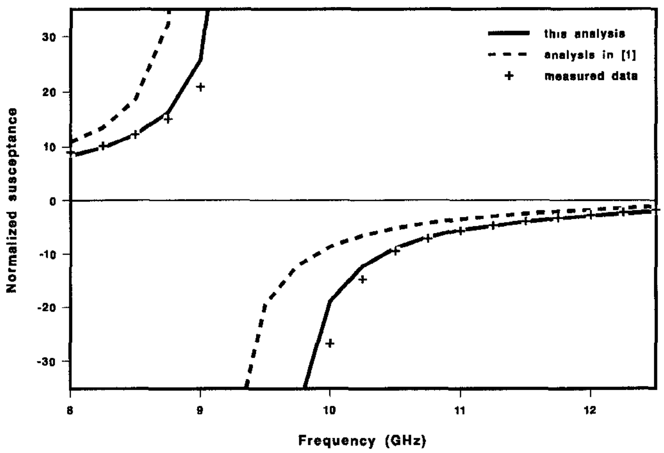

Fig. 4. Normalized susceptance of a strip with $w = 0.266$ and $d = 0.3543$ inch centered at 0.315 inch.

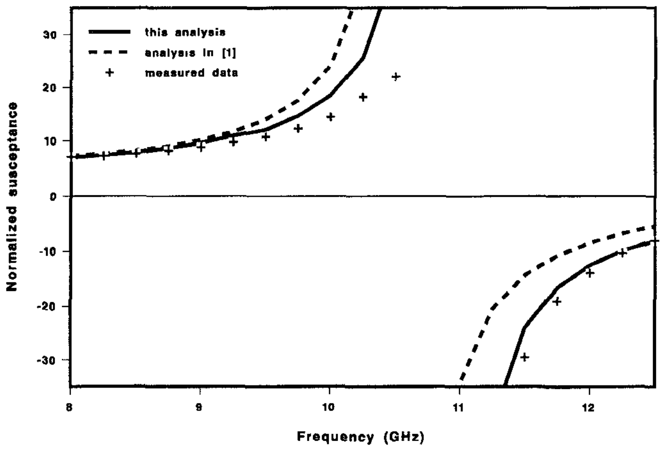

Fig. 5. Normalized susceptance of a strip with $w = 0.266$ and $d = 0.3543$ inch centered at 0.405 inch.

IV. Discussion

It was observed that near resonance, when the value of susceptance is large, the analysis with x-independent current is not accurate for the wide strip. Since the two terms in the denominator of the variational form are equal but of opposite signs at resonance [1], the denominator has a very small value near resonance. Since dividing by a small number is equivalent to multiplying a large number, numerical errors are magnified in this case, caused by the loss of significance [11]. Therefore, an accurate current distribution is needed for a wide strip operating near resonance, where the value of susceptance is large.

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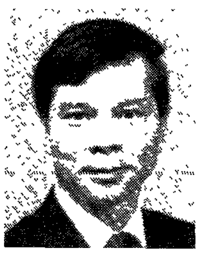

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